Stochastic Simulation Optimization: Optimal Computing Budget Allocation

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Decision Making Under Uncertainty

An Example:

Maximize or Minimize \( J(x) = E_w[ U(f(x, w)) ] \)
subject to system constraints and/or limited resources

where \( U(\cdot) \) reflects the utility/risk attitude of the decision maker
\( f \) is a vector of system performance measures
\( x \) is the vector of decision/control variables
\( w \) represents the uncertainty/variation

Assume \( x \in [x_1, x_2, x_3, \ldots, x_k] \), \( k \) is the number of alternative options
Simulation as An Engineering Design Tool

Advantages:

– Expand model flexibility
– Capable of including general form of utility functions
– Handle logical relationships, complex system dynamics, and precedence-based decision rules (e.g., discrete-event simulation).
– Provide excellent graphical interface
– Analyze system with consideration of uncertainty and variation (via Monte Carlo sampling).
– Web-based simulation can be accessed through internet using a web browser like Netscape or Internet Explorer, and can be run on any computer platform/operating system

Disadvantage:

– Efficiency is still a big concern
Function Evaluation: Deterministic vs. Stochastic

- **Deterministic**: an exact value $f(x)$
- **Stochastic**: a confidence interval obtained from multiple runs/samples/replications/evaluations

\[
\begin{align*}
\text{f(x)}
\end{align*}
\]

\[
\begin{align*}
\text{99\% Confidence Intervals}
\end{align*}
\]
Confidence Interval (C.I.)

\[
\frac{1}{N} \sum_{j=1}^{N} u(f(x_i, w_{ij})) \pm \frac{z\sigma}{\sqrt{N}}
\]

where

- \(z\) is the critical value for the standard normal distribution
- \(\sigma\) is the standard deviation of one simulation sample
- \(N\) is the number of simulation runs (samples)
Confidence Interval

\[ \frac{1}{N} \sum_{j=1}^{N} u( f(x_i, w_{ij}) ) \pm \frac{z \sigma}{\sqrt{N}} \]

Increase the number of simulation runs \((N)\)

99% Confidence Interval
Comparison of Two Alternative Designs

Which decision has higher expected return?

99% Confidence Interval

![Graph showing comparison of two alternative designs]
Multiple Comparison of

\(k\) alternative designs with estimation uncertainty

Correct Selection Probability

\[ P\{\text{CS}\} = P\{\text{Correct Selection of the True Best Design}\} \]

becomes bigger as we increase \(N\).

Simulation Efficiency

- \(N\) may be large in order to ensure a high \(P\{\text{CS}\}\)
- The number of alternative designs, \(k\), may not be small
- \(kN\) could be large → time-consuming
Simulation Procedures

Traditional Procedures

– Equal Simulation
  • All designs are equally simulated

– Rinott & Dudewicz Procedures: well-known procedures in IE/OR
  • Proportional to variance
  • \( N_i = c_i \sigma_i^2 \)

Our Intelligent Simulation

Optimal Computing Budget Allocation (OCBA)
An Intuitive Example - Maximization

99% Confidence Intervals

Equal Simulation

with the same number of total runs

Intelligent

⇒ Option 3 is better isolated
Marriage Problem

- 5 boys/girls to date
- Maximize your marriage utility function

**Equal Dating**

**Intelligent Dating**

⇒ #3 is standing out

with the same number of total dates
More General Case

99% Confidence Intervals

Equal Simulation with the same number of total runs

Intelligent ⇒ Option 3 is better isolated
Optimal Computing Budget Allocation

**OCBA**

(P1) Minimize the total number of simulation runs in order to achieve a desired simulation quality:

\[
\min_{N_1, \ldots, N_k} \left[ N_1 + N_2 + \ldots + N_k \right]
\]

s.t. \( P\{CS\} > P_{sat} \) (a satisfactory level)

(P2) Maximize the simulation quality with a given simulation budget:

\[
\max_{N_1, \ldots, N_k} P\{CS\}
\]

s.t. \( N_1 + N_2 + \ldots + N_k = T \) (total comp. budget)
OCBA Solution

Approach: Chen et al. (2000), Chen and Yücesan (2005)
- Common approximation to $P\{CS\}$
- Assume normality on noise and continuous $N_i$
- Consider asymptotic case
- Derive the allocation ratio based on KKT conditions

Asymptotically Optimal Solution

\[
\frac{N_i}{N_j} = \frac{(\sigma_i / \delta_{b,i})^2}{(\sigma_j / \delta_{b,j})^2} \quad \text{for } i \neq j \neq b
\]

\[
N_b = \sigma_b \sqrt{\sum_{i \neq b} (N_i^2 / \sigma_i^2)}
\]
Some Insights of OCBA Rule

inversely proportional to
the square of the signal to noise ratio

\[
\frac{N_2}{N_3} = \frac{\left( \frac{\delta_{1,3}}{\sigma_3} \right)^2}{\left( \frac{\delta_{1,2}}{\sigma_2} \right)^2}
\]
Simulation Run Allocation Using OCBA

Q. If T is increased to 16?
Alternatives

1

2

3

4

5

Simulation runs

Q. If T is increased to 22?
Numerical Testing: Allocation of Workers

- 31 workers: $C_1 + C_2 = 31$
- $C_1 \geq 11, C_2 \geq 11$
- $X = (C_1, C_2)$, 10 alternative designs
- Which has minimum average wait?

<table>
<thead>
<tr>
<th>Design</th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>19</td>
</tr>
<tr>
<td>**</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>11</td>
</tr>
</tbody>
</table>
Tested Allocation Procedures

Traditional Two-Stage Procedures

- Rinott Procedure
  - Specify $P^*$, e.g., 75%, a desired $P\{CS\}$ target
  - First Stage: $n_0$ runs for each to estimate variances
  - 2nd Stage: $N_i = \max(0, \left\lceil \frac{\sigma_i^2 h^2}{d^2} \right\rceil - n_0)$

Sequential Allocation

- Proportional To Variance (PTV):
  - Proportional to variance: $N_i = c_i \sigma_i^2$
- Equal Simulation
  - All designs are equally simulated
- OCBA
Numerical Results

- $P\{\text{CS}\} \nearrow$ as the computing budget $T$\nearrow

$\Rightarrow$ OCBA is more than 3 ~ 6 times faster in achieving 99% of $P\{\text{CS}\}$
Simulation Run Allocation \( (P_{CS}=99\%) \)

- OCBA
- Equal
- PTV

Design

Computing Budget Allocation

Design 1 2 3 4 5 6 7 8 9 10

- OCBA
- Equal
- PTV
More Designs, Higher Speedup

<table>
<thead>
<tr>
<th>Number of design alternatives, $k$</th>
<th>4</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>75</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speedup factor using OCBA vs. EA</td>
<td>1.75</td>
<td>3.42</td>
<td>6.45</td>
<td>12.8</td>
<td>16.3</td>
<td>19.8</td>
</tr>
</tbody>
</table>
Related Research

• **OCBA Types**
  – Glynn & Juneja (2004): non-Gaussian
  – Lee et al. (2004 & 2010): multiple objective functions
  – Fu et al. (2007): correlated sampling
  – Broadie et al. (2007): heavy-tailed distributions
  – He et al. (2007): opportunity cost
  – Frazier et al. (2008): correlated prior beliefs
  – Morrice et al. (2009): transient simulation

• **Other related works**
  – Chick and Inoue (2001ab): VIP procedures
  – Branke et al. (2007): procedures comparison and selection
STOCHASTIC SIMULATION OPTIMIZATION: An Optimal Computing Budget Allocation
by Chun-Hung Chen (George Mason University, USA & National Taiwan University, Taiwan) & Loo Hay Lee (National University of Singapore)

Stochastic Simulation Optimization addresses the pertinent efficiency issue via smart allocation of computing resource in the simulation experiments for optimization, and aims to provide academic researchers and industrial practitioners with a comprehensive coverage of OCBA approach for stochastic simulation optimization. Starting with an intuitive explanation of computing budget allocation and a discussion of its impact on optimization performance, a series of OCBA approaches developed for various problems are then presented, from the selection of the best design to optimization with multiple objectives. Finally, this book discusses the potential extension of OCBA notion to different applications such as data envelopment analysis, experiments of design and rare-event simulation.