OR/STAT 645 Midterm Examination -- 2009

1. For a branching process, calculate $\pi_0$ when
   (a) (8) $P_0 = 0.3$, $P_1 = 0.3$, $P_2 = 0.4$.
   (b) (7) $P_0 = 1/2$, $P_1 = 1/4$, $P_2 = 1/8$, $P_3 = 1/8$.

2. A store opens at 8 AM. From 8 until 10 customers arrive at a Poisson rate of four an hour. Between 10 and 12 they arrive at a Poisson rate of eight an hour. Please give
   - $E$ = the expected number of arrivals, and
   - $P_0$ = the probability of no arrival
during each of the following time intervals.
   (a) (5) Between 8:30 and 9:30 AM
   (b) (5) Between 9:30 and 10:30 AM
   (c) (5) Between 10:30 and 11:30 AM
   (d) (5) Between 8:00 and 12:00 Noon

3. Taxis arrive at an airport in the morning rush hours according to Poisson process with rate of four per minute. The number of passengers a taxi carries is 1 or 2 with probability $2/3$ and $1/3$.
   (a) (8) What is the expected number of arriving passengers in two hours?
   (b) (7) What is the variance for the number of arriving passengers in one hour?

4. Two light bulbs are simultaneously put on a life test. Suppose the lifetime for bulb #1 is exponentially distributed with mean 10 hours and #2 is exponentially distributed with mean 15 hours. Assume these two bulbs are independent. The test will end when both bulbs fail.
   (a) (5) What is the expected time for the first failure?
   (b) (5) What is the variance of the time for the first failure?
   (c) (10) On average, how long will the test take?
5. Suppose we classify the weather into three possible states: Sunny, Cloudy, and Rainy. If today is a sunny day, tomorrow will be sunny with probability 0.6, cloudy with probability 0.3. If today is cloudy, tomorrow will be cloudy with probability 0.5, sunny with probability 0.3. If today is rainy, tomorrow will be rainy with probability 0.4, cloudy with probability 0.3.

(a) (8) Calculate the proportion of days it rains.

(b) (7) If it rains today, what is the probability that it will rain the day after tomorrow?

6. Students arrive at Johnson Center at a Poisson rate of 20 per minute. 20% of them are freshmen in college. Let T denote the interarrival time between two arriving freshmen.

(a) (7) What is the statistical distribution for T (including all the parameters)?

(b) (8) Give a rigorous proof for your part (a). Hint: derive the cdf of T.