1. Answer all questions
2. Exam time is 2 hours.
3. The exam is open book open notes.
4. Laptop or other computers are not allowed.
5. Show your work

1. 10 points. Determine whether the following vectors are linearly independent:

\[(1, 0, 1), (2, -1, 0) \text{ and } (0, 2, -4)\]

2. Consider the following system of equations:

\[
\begin{align*}
2x_1 + x_2 &= 3 \\
-2x_1 + 2x_3 &= -2 \\
-4x_1 + x_2 + 7x_3 &= -2
\end{align*}
\]

(a) Solve the problem via Gaussian elimination

(b) Compute the determinant of the system matrix \(A\).

3. Use Gaussian elimination to determine whether the following linear systems have a solution, and whether the solution is unique. When several solutions exist, find all solutions

(a)

\[
\begin{align*}
-2x_1 + 8x_2 + 4x_3 &= 4 \\
x_1 - 4x_2 - 2x_3 &= 3
\end{align*}
\]

(b)

\[
\begin{align*}
-2x_1 + 8x_2 + 4x_3 &= 4 \\
x_1 - 4x_2 - 2x_3 &= -2
\end{align*}
\]

4. Compute the eigenvalues and eigenvectors of the following matrix

\[
A = \begin{pmatrix}
-3 & 4 \\
2 & -1
\end{pmatrix}
\]

5. Compute the vector of first partial derivatives (the gradient) and the matrix of second partial derivatives (the Hessian) for the function

\[f(x) = e^{2x_1+3x_2} + 2x_1x_2\]

6. Solve the following differential equations:

(a)

\[y' + 3y = 0\]

1
7. Use partial fraction decomposition to express the rational function below as the sum of rational functions with linear denominators.

\[ \frac{4s}{s^2 + 2s - 3} \]

(b) Use the decomposition you obtained to determine

\[ \int \frac{4s}{s^2 + 2s - 3} \, ds \]

(c) Use the decomposition you obtained to determine the inverse Laplace transform of

\[ F(s) = \frac{4s}{s^2 + 2s - 3}, \]

that is find the function \( f(x) \) whose Laplace transform satisfies \( L\{f(x)\} = F(s) \).

8. A jar contains 20 red jelly beans, 10 green jelly beans, and 5 black jelly beans. You stick your hand into the jar and scoop out a handful of 6 jelly beans.

(a) What is the probability that 3 of them will be black?

(b) What is the probability that your handful also includes some red jellybeans?